5.8. MEASUREMENT OF THE MODULUS OF ELASTICITY

Purpose of experiment

To determine the modulus of elasticity for different materials and to examine bending ratio dependence on different sample parameters and applied forces.

Tasks of experiment

- Determine the modulus of elasticity for steel, aluminium, brass and/or human bone samples.
- Examine the bending of flat bars as a function
  - of the force,
  - of the thickness, at constant force,
  - of the width, at constant force,
  - of the distance between the support points at constant force.

Theoretical topics

- Properties of materials.
- Stress, deformation.
- Hooke’s Law.
- Elasticity (Young’s) modulus.

Equipment and materials

Dial gauge 10/0.01 mm, holder for dial gauge, flat bar set and/or bone tissue sample, knife-edge with stirrup, bolt with knife-edge, weight holder f. slotted weights, spring balance 1 N, tripod base, support rod, square, l = 250 mm, support rod, square, l = 630 mm, right angle clamp, slotted weight, 10 g, black, slotted weight, 50 g, black, measuring tape, l = 2 m.

Theoretical part

There is a variety of ways of classifying the different parts of the human body from a mechanical perspective. Body components, for example, can be either passive or active. Passive components, such as bones and tendons, respond to outside forces. Active elements such as muscles, generate forces. This division, however, is not perfect. Muscles are indeed active elements, but they also have some properties of passive components, and when they are modeled, both their active and passive properties must be included. Passive elements respond to applied stresses (forces/area) in a complex way. Their response to forces can be either independent or dependent with regard to time, meaning that the component can respond only to currently applied forces or to both current forces and forces applied earlier.

A passive response is most simply exemplified by linear or Hookean behavior, in which the properties of the material behave exactly like that of an ideal harmonic oscillator spring. Deformations are linear with the applied forces and stresses and the response is independent of time. All the potential energy stored in such media can be extracted. Bones and tendons are fairly well (but not perfectly) modeled as such elastic media. The elastic nature of tendons makes them...
very important in energy storage and retrieval during motion. Some materials systems, such as metal springs and bones, behave similarly under tension and compression. Others, such as cartilage and tendons, do not.

This difference is due to the fact that no material is perfectly harmonic. Most materials deviate from perfectly harmonic behavior for large applied forces and large deformations. A material can deviate from a harmonic oscillator (in classical mechanics, harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force proportional to the displacement) dependence with the deformation depending nonlinearly on force or stress, and yet this deformation can still be reversible. This means that the material returns to its initial state when the stress is removed both in the linear and nonlinear parts of this elastic regime or region (Fig. 5.8.1). For even larger stresses, the material is no longer elastic because it undergoes plastic deformation, which is irreversible. This means that the material never returns to the same size or shape when the stress is removed. For even larger stresses, there is fracture. One glaring example is the fracture of bones.

If a rod of some material is subjected to a deformation along its axis, then it will be expected to change in length. A deformation/displacement curve can be drawn based on experimental data and the displacement in terms of extension per unit length can be described. This displacement is called strain ($\varepsilon$), and the deformation in terms of force per unit area, is called stress ($\sigma$). The deformation/displacement curve can be redrawn as a stress/strain curve as shown in Figure 5.8.1, which shows a realistic stress–strain relation. This should be independent of the dimensions of the bar.

The shape of the stress/strain curve illustrated in figure 5.8.1 is typical of many engineering materials, and particularly of metals and alloys. In the context of biomechanics it is also characteristic of bone. There is a linear portion between the origin O and the point $Y_p$. In this region the stress is proportional to the strain:

$$\sigma = E\varepsilon,$$

where $\sigma = F/S$ and $\varepsilon = \Delta l/l_0$. (5.8.1)

The constant of proportionality $E$ is called Young’s modulus or modulus of elasticity. The linearity of the equivalent portion of the stress/strain curve is known as Hooke’s law. There is elastic Hookean behavior up to the point P, the proportional limit. The slope up to this stress is constant, the Young’s modulus $E$. The higher the $E$, the stiffer or the less compliant the material (Fig. 5.8.1). At higher stresses, the stress–strain relation is nonlinear. Up to the elastic
limit, denoted by $E_L$, the object returns to its initial length when the stress is removed and there is no permanent deformation. In the linear and nonlinear elastic regimes, the stretched bonds relax totally and there is no rearrangement of atoms after the force is released. **Permanent or plastic deformation** occurs when stresses are beyond the elastic limit, and the length and shape of the object are different after the stress is removed. The **yield point or limit**, denoted by $Y_p$, is at a stress somewhat higher than the elastic limit; above it much elongation can occur without much increase in the deformation. Typically the point $Y_p$ represents a critical stress in the material. (Some do not distinguish between the elastic limit and the yield point.) Because it is often difficult to determine, the yield point is usually estimated by the intersection of the stress–strain curve with a line parallel to the linear part of the stress–strain curve, but with an intercept set at a strain of 0.2% (or 0.002). The yield point occurs at the yield stress (or strength), $Y_S$. For tension, the material remains intact for larger stresses until the **ultimate tensile stress** (UTS), which is also called the **tensile strength** (TS) or, less commonly, the **tensile breaking strength** (TBS). The larger the breaking strength, the stronger is the material. Application of this stress leads to fracture at point $F$, which occurs at a strain called the **ultimate strain** or the **ultimate percent elongation** (UPE).

The curve in Figure 5.8.1 represents the response to **Tensile deformation**. But there are other types of deformation: Compression, Bending, Shear, Torsion.

When the member is subjected to a simple uniaxial tension or compression, the stress is just the deformation force divided by the cross-sectional area of the member at the point of interest. Whether a tensile stress is sustainable can often be deduced directly from the stress/strain curve. Many structures must sustain bending moments as well as purely tensile and compressive deformations. It will be seen that a bending moment causes both tension and compression, distributed across a section. A typical example is the femur, in which the offset of the force applied at the hip relative to the line of the bone creates a bending moment. One-third of the weight is in the legs themselves, and each femur head therefore transmits one-third of the body weight.

Shear stresses can arise when tractile forces are applied to the edges of a sheet of material. Shear stresses represent a form of biaxial loading on a two-dimensional structure. It can be shown that for any combination of deformations there is always an orientation in which the shear is zero and the direct stresses (tension and/or compression) reach maximum and minimum values. Conversely, any combination of tensile and compressive (other than hydrostatic) deformations always produces a shear stress at another orientation. In the uniaxial test specimen the maximum shear occurs along lines orientated at $45^\circ$ to the axis of the specimen. Pure two-dimensional shear stresses are in fact most easily produced by loading a thin-walled cylindrical beam in torsion.

The most common torsional failure in biomechanics is a fracture of the tibia, often caused by a sudden arrest of rotational motion of the body when a foot is planted down. This injury occurs when playing games like soccer or squash, when the participant tries to change direction quickly. The fracture occurs due to a combination of compressive, bending and torsional deformations. The torsional fracture is characterized by spiral cracks around the axis of the bone.

**Material Components of the Body.** More generally, there are four categories of tissues:

1. **Epithelial tissue** covers the body and lines organs or secretes hormones. It has closely packed cells, little intercellular material, nerves, and no blood vessels (and so it is avascular).

2. **Connective tissue** includes bone, cartilage, dense connective tissue (such as ligaments and tendons), loose connective tissue – such as “fat” – and blood and lymph vascular tissue. Most connective tissue has nerves and scattered cells in a background called a matrix. There are many blood vessels in bone and at the periphery of the menisci – and so they are highly vascularized, but tendons, ligaments, and (the bulk of) cartilage are not. The matrix consists of fibers and ground substances. The fibers include collagen fibers (made of the protein collagen) that are tough and flexible; elastic fibers (made of the protein elastin) that are strong and stretchable; and reticular, web-like fibers. The ground substance includes cell adhesion proteins to hold the tissue together and proteoglycans to provide firmness. **Epithelial membranes** consist of epithelial and connective tissue.

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These line the body (skin (cutaneous membrane)), internal organs (serous membranes of the heart (pericardium), lungs (pleura), and abdominal structures (peritoneum)), cavities that open to the outside world (mucous membranes of the nasal cavity, and the respiratory, gastrointestinal, and urogenital tracts), and cavities at bone joints (synovial membranes).

3) Nervous tissue, for body control, consists of neurons to transmit electrical signals and neuroglia (or glial cells) to support the neurons, by insulating them or anchoring them to blood vessels.

4) Muscle tissue controls movement, and includes passive components (such as in the connective tissue) and active, motor-like components.

Bones provide a structural framework to attach muscles and organs, enable movement through the attachment of muscles, provide physical protection of organs (such as the skull for the brain and the rib cage for the lungs), store minerals (calcium and phosphorus) and some fats (in the yellow marrow), and produce red blood cells (in the red marrow).

Bone is a complex composite material, with living and nonliving matter, containing organic and calcium-based inorganic components. The living matter includes the cells osteoblasts and osteoclasts, which, respectively, make new bone and resorb (erode) existing bone, and osteocytes, which are former osteoblasts buried in bone they have made. Excluding water, the nonliving matter of bone is 40% by weight (60% by volume) collagen and 60% by weight (40% by volume) calcium hydroxyapatite (Ca10(PO4)6(OH)2). The ∼5nm × 5nm × 40nm rod or plate crystals with hexagonal symmetry of the ceramic-like calcium hydroxyapatite are bound by the elastomer like collagen. The inorganic ceramic component gives compact bone its large strength (a large elastic constant $E$) and a large ultimate compressive stress (UCS). The collagen component makes bone much more flexible than a ceramic and much more stable under tension and bending. About 1% of the organic component is proteoglycans (mucopolysaccharides). About 25% of the volume of bone is water, ~60% of which is bound to the collagen. Spongy (or trabecular) bone has voids with lateral dimensions of 50–500 μm.

The osteoid contains collagen, a fibrous protein found in all connective tissues. It is an elastic material ($E \approx 1.2$ GPa) that serves as a matrix and carrier for the harder and stiffer mineral material. The collagen provides much of the tensile strength (but not stiffness) of the bone. Deproteinized bone is hard, brittle and weak in tension, like a piece of chalk.

The mineral salts give the bone its hardness and its compressive stiffness and strength. The stiffness of the salt crystals is about 165 GPa, approaching that of steel. Demineralized bone is soft, rubbery and ductile.

The skeleton is composed of cortical (compact) and cancellous (spongy) bone, the distinction being made based on the porosity or density of the bone material. The division is arbitrary, but is often taken to be around 30% porosity (Figure 5.8.2). Cortical bone is found where the stresses are high and cancellous bone where the stresses are lower (because the deformations are more distributed), but high distributed stiffness is required. The aircraft designer uses honeycomb cores in situations that are similar to those where cancellous bone is found.

Cortical bone is hard and...
has a stress/strain relationship similar to many engineering materials that are in common use. It is anisotropic, and the properties that are measured for a bone specimen depend on the orientation of the deformation relative to the orientation of the collagen fibres. Furthermore, partly because of its composite structure, its properties in tension, in compression and shear are rather different. In principle, bone is strongest in compression, weaker in tension and weakest in shear. The strength and stiffness of bone also vary with the age and sex of the subject, the strain rate and whether it is wet or dry. Dry bone is typically slightly stiffer (higher Young’s modulus) but more brittle (lower strain to failure) than wet bone. Some of the mechanical properties of the femur are summarized in Table 5.8.1.

Ligaments and tendons are dense connective tissue with a dense network of fibers, with few cells and little ground substance. Ligaments are tough bands of fibrous connective tissue. They are 55–65% water and 35–45% dry matter, which consists of 70–80% collagen (mostly type I), 10–15% elastin, and a small amount, 1–3%, of proteoglycans. In each of these soft materials, the collagen gives tensile strength, while the elastin gives elastic properties, which are more important in ligaments than in tendons.

Table 5.8.1. Mechanical properties of bone.

<table>
<thead>
<tr>
<th>Bone</th>
<th>Tension</th>
<th>Compression</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$, MPa</td>
<td>$\varepsilon$, %</td>
<td>$E$, GPa</td>
</tr>
<tr>
<td>Femur</td>
<td>124</td>
<td>1.41</td>
<td>17.6</td>
</tr>
</tbody>
</table>

Methodology

In this labwork, the modulus of elasticity is determined from the bending deformation and the geometric data of the bar. The device for its determination is shown in Figure 5.8.3. On vertical stands A1 and A2, triangular prisms P1 and P2 are fixed. One of the flat bars is placed on these prisms and must be positioned accurately on the two supporting knife edges having the possibility to move in $x$ and $y$ direction. The coupling is put on the bar and a holder with weights is attached to the coupling.

The coupling is placed halfway between the triangular prisms. Above the bar is a consolidated rod,
to which is attached a micrometer B. The geometric data of the set-up and the bars must be recorded several times or at different positions. If a bar of height \( b \) and width \( a \), supported at both ends by supports (separated by a distance \( L \)), is subjected to a force \( F_y \) acting at its centre, it behaves like a bar supported in the middle, its two ends being subjected to a force \( F_y/2 \) in the opposing direction. In order to express the bending \( \lambda \) as a function of the modulus of elasticity \( E \), an element of volume should first be considered (Fig. 5.8.4):

\[
dV = dx \cdot a \cdot b,
\]

(5.8.3)

the upper layer of which is shortened on bending, and the bottom layer lengthened. The length of the central layer remains unchanged (neutral fibre). In Fig. 5.8.4, I and II denote the sides before and after deformation.

Using the symbols given in Figure 5.8.4., the deflection is:

\[
d\lambda = x \cdot d\phi = \frac{2hx}{b}.
\]

(5.8.4)

The elastic force \( dF_x \), which produces the extension \( dl \), is

\[
\frac{dF_x}{ds} = E \frac{dl}{dx}
\]

(5.8.5)

where \( ds = ady \) is the area of the rotated layer.

The force produces a torque:

\[
dT_z = ydF_x = \frac{2Eah}{b \cdot dx} y^2 dy
\]

(5.8.6)

The sum of these torques produced by the elastic forces must be equal to the torque produced by the external force \( F_y/2 \):

\[
\frac{Eahb^2}{6dx} = \frac{F_y}{2} x
\]

(5.8.7)

from which is obtained

\[
d\lambda = \frac{6F_yx^2}{Eab^3} \, dx
\]

(5.8.8)

and, after integration, the total deflection:

\[
\lambda = \frac{1}{4} \left( \frac{L}{b} \right)^3 \cdot \frac{1}{a} \cdot \frac{F_y}{E}
\]

(5.8.9)
Procedures

1. With the ruler, measure the length of the sample (its length is understood as the distance between the supports) and with a caliper or micrometer measure the width b and thickness a. For each quantity, the measurements should be carried out at least three times at different locations on the sample and the average values calculated.

2. Enter the data in Table 1.

Table 1.

<table>
<thead>
<tr>
<th>Sample No and title</th>
<th>(a_i), m</th>
<th>(a_{avr}), m</th>
<th>(b_i), m</th>
<th>(b_{avr}), m</th>
<th>(l_i), m</th>
<th>(l_{avr}), m</th>
</tr>
</thead>
</table>

3. The sample is placed on the supports P1 and P2 (Figure 5.8.3.) The coupling is attached to it. A micrometer is attached to the middle of the coupling so that it shows ~ 2 mm deflection. Set the initial value \(n_0\) of the micrometer.

4. Carefully put the weight holder with the slotted weight \(m_1\) on the coupling, record the reading of the micrometer \(n\); determine the deflection \(\lambda = n - n_0\).

5. Remove the weight holder and place another weight on it. Once again, place the weight holder with slotted weight \(m\) on the coupling. Repeat step 4 for 5-6 different weights \(m_i\).

6. Find the deformation force \(F = mg\) (here \(- g\) is free fall acceleration) and calculate the ratio \(F/\lambda\). Calculate the mean value of the ratios.

7. Calculate the Young’s (elasticity) modulus \(E\) (factor is \(\frac{1}{4}\), since both ends of the sample were placed on supports):

\[
E = \frac{Fl^3}{4\lambda ba^3}.
\]  

(5.8.10)

8. All measurements and calculations should be repeated for other samples.

9. In Table 2, enter the data from the results of the measurements and calculations.

Table 2.

<table>
<thead>
<tr>
<th>Sample No and title</th>
<th>(m), kg</th>
<th>(F), N</th>
<th>(\lambda), m</th>
<th>(F/\lambda), N/m</th>
<th>((F/\lambda)_{avr}), N/m</th>
<th>(E), Pa</th>
</tr>
</thead>
</table>

10. Plot the graph \(\lambda = \lambda (F)\) for each sample.

**Note.** The results should be compared with the modulus values presented in Table 3.

Table 3.

<table>
<thead>
<tr>
<th>Material</th>
<th>Sizes, mm</th>
<th>Modulus of elasticity (E), N/m(^2) (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bone (av.)</td>
<td></td>
<td>(1 \times 10^{10})</td>
</tr>
<tr>
<td>Wood (oak)</td>
<td></td>
<td>((9,1+11,8) \times 10^{10})</td>
</tr>
<tr>
<td>Steel</td>
<td>(10 \times 1,5)</td>
<td>(2,059 \times 10^{11})</td>
</tr>
</tbody>
</table>
# MEASUREMENT OF THE MODULUS OF ELASTICITY

<table>
<thead>
<tr>
<th>Material</th>
<th>Dimensions</th>
<th>Modulus of Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>10 × 2</td>
<td>2.06310^{11}</td>
</tr>
<tr>
<td>Steel</td>
<td>10 × 3</td>
<td>2.17110^{11}</td>
</tr>
<tr>
<td>Steel</td>
<td>15 × 1,5</td>
<td>2.20410^{11}</td>
</tr>
<tr>
<td>Steel</td>
<td>20 × 1,5</td>
<td>2.11110^{11}</td>
</tr>
<tr>
<td>Aluminium</td>
<td>10 × 2</td>
<td>6.70210^{10}</td>
</tr>
<tr>
<td>Brass</td>
<td>10 × 2</td>
<td>9.22210^{10}</td>
</tr>
</tbody>
</table>

References:

1. Irving P. Herman, Physics of the human body, Berlin; Heidelberg: Springer (2007).